Corrigendum of the paper

"On the proper analytical expression for the sky-view factor and the diffuse irradiation of a slope for an isotropic sky"

Introduction

In our paper [1] we discussed the difference between the geometric portion of visible sky and the ratio between the diffuse irradiation on a tilted (tilt angle τ) and on a horizontal receiving surface. Our aim was to emphasis the difference between them as both are usually described by the same term: sky-view factor – thus we denoted the first as SVF and the second one we named as diffuse tilt factor (DTF). In consideration of diffuse solar irradiance we included also ground reflectivity R and the proportion between the diffuse and the global irradiance k. Unfortunately, although making the clear division between SVF and DTF, the mistake in one of our calculations caused the false final result for DTF that led us also to some inappropriate discussions. We correct them here and apologize for the additional confusion to the matter.

Corrections

Performing integrations (orig. eq. 5) to obtain diffuse tilt factor (DTF) we overlooked the fact that the elevation angle of horizon τ' for a tilted plane with a tilt angle τ depends on azimuth φ. This follows from spherical trigonometry:

$$\mathrm{tg}\,\tau' = -\cos\varphi\ \mathrm{tg}\,\tau, \text{ or alternatively } \cos\tau' = \cos\tau / \sqrt{(1 - \sin^2\varphi \sin^2\tau)}$$

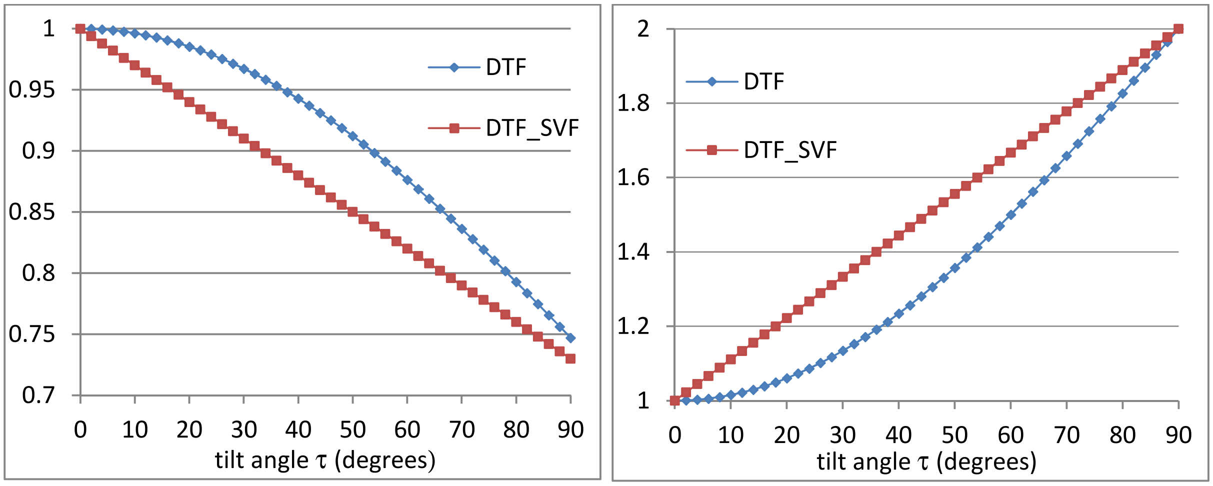
So the two integrals ignoring this dependence are not generally valid. The general result for Eq. 5, valid for slopes without any additional obstacles in horizon is thus (see appendix):

$$E_{diff\_tilt} = \ldots = L_{sky}\,\pi\,[\cos^2(\tau/2) + R/k \sin^2(\tau/2)] \qquad \text{(corr. eq. 5)}$$

and the proper DTF is accordingly the one which we denoted as $DTF_{L\&J}$:

$$DTF = E_{diff\_tilt} / E_{diff\_hz} = \cos^2(\tau/2) + R/k \sin^2(\tau/2) = (1 + \cos\tau)/2 + R/k\,(1 - \cos\tau)/2. \qquad \text{(corr. eq. 6)}$$

The first of the two terms, describing the part of irradiation from the sky $\cos^2(\tau/2) = (1 + \cos\tau)/2$ is the one that Kondtat'ev [2], Liu and Jordan [3,4] and others use for diffuse irradiation of the slope from the sky. Our previously computed DTF (orig. eq. 6) is valid only for a special case when a tilted plane is additionally obscured with a semi-circular obstacle with radius r, of a height $h = r\,\mathrm{tg}\tau$.



Figs. 4 & 5 corr.: The comparison between the proper DTF and DTF_SVF using the geometrical SVF for R = 0.2 and k = 0.5 (left – fig. 4 corr.) and for R= 0.9 and k = 0.3 (right – fig. 5 corr.).

Conclusion

Geometrical sky-view factor SVF and the diffuse tilt factor DTF describe different characteristics of the receiving surface. SVF considers only integration of solid angles. It describes the ratio between the visible part of the sky and the whole half-dome of the sky. Its only proper expression is $(\pi - \tau)/\pi$ [1,5].

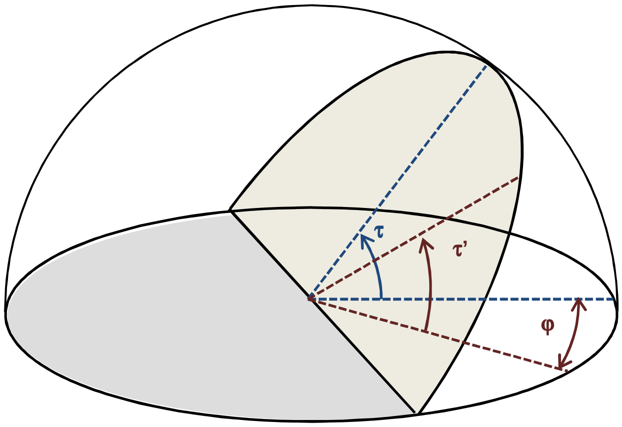
DTF relates to the amount of diffuse irradiation of the slope in comparison to the diffuse irradiance of the horizontal surface. It includes also the cosine of impact angle into integration of solid angles. For a slope without any additional obstacles on horizon it is defined as: $DTF = \cos^2(\tau/2) + R/k \sin^2(\tau/2)$ [2,3,4].

Using SVF instead of the first term $\cos2\tau/2$ of DTF is not appropriate for diffuse irradiation purposes and vice-versa: it is not appropriate using this first term to estimate a portion of the visible sky.

References

[1] Rakovec J, K Zakšek. On the proper analytical expression for the sky-view factor and the diffuse irradiation of a slope for an isotropic sky. Renewable Energy 37 (2012) 440-444.
[2] Kondrat'ev KJ, Pivovarova ZI, Fedorova MP. Radiacioniirezhimnaklonnihpoverhnostej. Leningrad: Gidrometeoizdat; 1978.
[3] Liu BYH, Jordan RC. Daily insolation on surfaces tilted towards the equator.Trans ASHRAE 1962;67:526-41.
[4] Liu BYH, Jordan RC. The long-term average performance of flat-plate solarenergy collectors. Sol Energ 1963;7:53-74.
[5] Tian YQ, Davies-Colley RJ, Gong P, Thorrold BW. Estimating solar radiation onslopes of arbitrary aspect. Agric For Meteorol 2001;109:67-74.

Appendix



Integration for the second part of original eq. 5.

$$\frac{1}{2}\int_{\pi/2}^{3\pi/2} 1-\frac{\cos^2\tau}{1-\sin^2\tau\sin^2\varphi}\,d\varphi = \qquad \sin^2\tau = a,\ \cos^2\tau = 1-a$$

$$= \frac{1}{2}\int_{\pi/2}^{3\pi/2} 1-(1-a)\frac{1}{1-a\sin^2\varphi}\,d\varphi =$$

$$= \frac{1}{2}\left[\varphi-(1-a)\frac{1}{\sqrt{1-a}}\,arctg\left(tg\varphi\sqrt{1-a}\right)\right] = * \qquad \mathrm{tg}\frac{\pi}{2} = \mathrm{tg}\frac{3\pi}{2} = \pm\infty$$

$$arctg\left(\pm\infty\sqrt{1-a}\right) = 0 \text{ or } \pi \text{ (we take } \pi\text{)}$$

$$* = \frac{1}{2}\left[\frac{3\pi}{2}-\frac{\pi}{2}-\pi\sqrt{1-a}\right] =$$

$$= \frac{\pi}{2}\left[1-\sqrt{1-a}\right] =$$

$$= \frac{\pi}{2}\left[1-\sqrt{1-\sin^2\tau}\right] =$$

$$= \frac{\pi}{2}\left[1-\cos\tau\right] = \pi\sin^2\frac{\tau}{2}$$